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SPARSE SAMPLING FOR MAGNETIC RESONANCE IMAGING

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Abstract: *Functional Magnetic Resonance Imaging (MRI) and other dynamic MRI applications, require rapid acquisition to measure dynamic processes changes. Experimental data are collected in the k-space by following different trajectories to cover the whole space. Complete data acquisition needs several minutes: the reduction of the number of collected trajectories allows proportional acquisition time reduction but undersampling occurs, producing artefacts. In what follows, MRI sparse sampling acquisition and reconstruction methods are overviewed. In particular, sparse methods are grouped in two classes: the first contains methods in which the sampling scheme is independent of the sample shape, the most important is Compressed Sensing (CS); the other contains methods that adapt their sampling pattern, by modifying the acquisition trajectories (both in number and directions) during the acquisition, to the sample internal structure. In this second class, an emerging set of methods, hybrid forms of adaptive CS, are included and discussed. The current paper clarify the importance of using adaptive CS strategies in MRI to reduce acquisition time and undersampling artefacts and to improve the signal to noise ratio (SNR) of the resulting image.*

1 INTRODUCTION

In conventional MRI, the number of collected data for each image is determined by spatial resolution requirements and by the Nyquist's theorem constraints to obtain the desired resolution in the aliasing-free, fully-sampled, image. Recently, MRI has developed toward dynamic imaging opening up several new applications such as monitoring of contrast agent dynamics [1], mapping of human brain neural activity based on blood oxygenation level-dependent (BOLD) imaging contrast [2], MR-guidance of biopsies [3-5], monitoring of ablations or guidance of intravascular procedures [6-12], and real-time visualization of cardiac motion [13,14]. Although these developments are promising, they are limited by the compromise between temporal and spatial resolution. To improve temporal resolution, sampling strategies use "undersampling". The term undersampling indicates that the Nyquist criterion is not satisfied, at least in some regions of the sampling domain, and images are reconstructed by using a number of samples lower than that theoretically required to obtain a fully-sampled image. The sampling domain for MRI is called *k*-space, that is an array of complex numbers, each defined in a given spatial position (*kx*,*ky*,*kz*), obtained directly from the MRI signals, whose values correspond to spatial frequencies of the MRI image. The *k*-space data and the resulting image are related through a Fourier Transform. The sampling trajectories used in MRI are mainly those reported in Figure 1.

Normally undersampling implies image artefacts in the form of aliasing or streak structures. Parallel imaging methods such as simultaneous acquisition with spatial harmonics (SMASH) [15], sensitivity encoding (SENSE) [16], and generalized autocalibrating partially parallel acquisition (GRAPPA) [17] also can be thought as parallel undersampling methods from multiple RF coils and receivers, in which a complete image is reconstructed by merging the partial information collected by different receivers.

Time required to fully sample 3D Cartesian *k*-space is relatively long. Alternative non-Cartesian trajectories can provide faster *k*-space coverage and more efficient gradients usage. When very fast volume coverage is required, undersampling strategies can be combined with non-Cartesian trajectories for further reduction of the scan time.

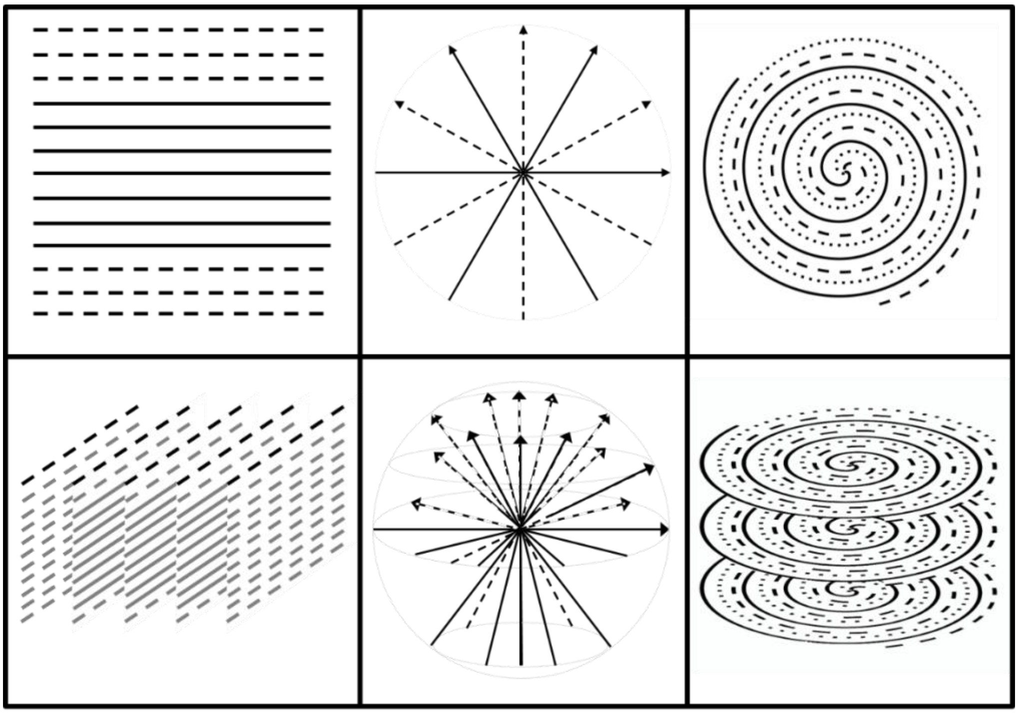

Figure 1: Cartesian (first column), radial (second column) and spiral (third column) sampling in 2D (first row) and in 3D (second row) used in MRI. Dashed lines indicate missing trajectories in case of undersampling.

Undersampling can influence the resulting image in different ways depending on the *k*-space covering paths (see Figure 1). This implies that it can occur differently in different *k*-space regions. All the sampling/reconstruction methods afford the same problem: to reconstruct an image, suppose a 2D image $f(x, y)$, starting from some collected sparse samples of its Fourier coefficients, $\mathrm{F}|_{\Omega}$ while violating the Nyquist rate. In this paper, recent sparse sampling and restoration strategies for solving the previous problem are reported, by making a distinction between methods that reduce artefacts independently of the sample shape and structure (acquisition trajectories independent of the sample) and those that adapt the acquisition trajectories (both in number and in directions) to the sample shape (during acquisition, the information content of the collected data is used to infer the "most informative" future trajectories). The current paper speculates on the importance of using adaptive sampling strategies in MRI, considering that in the last years some doubts have arisen regarding the real usefulness of these methods [18].

2 SAMPLE-INDEPENDENT SPARSE METHODS

This first group includes methods for which redundancy is an implicit property of the image, or of its transform in some space, but not directly used for driving data acquisition: acquisition uses an undersampling pattern which is independent of the underlying structured shape of the image.

During the last few years, the emerging theory of compressive (or compressed) sensing (CS) [19-23] has offered great insight into both when and how a signal may be recovered to high accuracy (or, for some instances, exactly) even when sampled significantly below the Nyquist rate.

CS requires the measurement of a relatively small number of "random" linear combinations of the signal values (much smaller than the number of signal samples nominally defining it). However, because the underlying signal is compressible, the nominal number of signal samples is a gross overestimate of the "effective" number of "degrees of freedom" of the signal. As a result, the signal can be reconstructed with good accuracy from relatively few measurements by a convex constrained optimization procedure. In MRI the sampled linear combinations are simply individual Fourier coefficients (*k*-space samples) and CS can be used. In that setting, CS is claimed to be able to make accurate reconstructions from a small subset of *k*-space, rather than an entire *k*-space grid. The original paper by Candès et al. [19] is dedicated at random undersampling of Fourier coefficients, that is the practical situation of MRI [22]. In order to reconstruct a complete image from the undersampled problem, the simpler strategy assumes that the Fourier coefficients at all of the unobserved frequencies are zero (thus reconstructing the image of "minimal energy" under the observation constraints). This method does not perform very well because the reconstructed image has severe non local artefacts caused by angular undersampling [19]. A good reconstruction algorithm, it seems, would have to guess the values of the missing Fourier coefficients, i.e. to interpolate $F(k_x, k_y)$. However, the prediction of Fourier coefficients from their neighbours are very delicate, due to the global and highly oscillatory nature of the Fourier transform. The prediction can be more efficiently done through convex optimization. To recover f from partial Fourier samples, a solution f^* is found for the optimization problem

$$min\|g\|_{TV} \text{ subject to } G(\boldsymbol{k}) = F(\boldsymbol{k})\ \forall \boldsymbol{k} \in \Omega \tag{1}$$

where $\|g\|_{TV}$ represents the total-variation norm of a 2D object g that, for discrete data $g(x,y)$, $0 \leq x\ \ , y \leq N-1$, has the following form

$$\|g\|_{TV} = \sum_{t_1,t_2} \sqrt{|D_1 g(x,y)|^2 + |D_2 g(x,y)|^2} \tag{2}$$

and $D_1 g = g(x,y) - g(x-1,y)$, $D_2 g = g(x,y) - g(x,y-1)$

As it is, this technique allows just to reduce artefacts with respect to zero filling of the missing Fourier coefficients (with the exception of the first example reported in [19]). For an accurate reconstruction, also in presence of undersampling, it is necessary that:

1. the image has a sparse representation in a known transform domain (i.e., it must be

compressible by a transform coding: for an n-dimensional object f, if s is the sparsity term, that is the number of significant terms of f in some domain, it must be $s<<n$).
2. the artefacts caused by k-space undersampling are incoherent (noise like) in the sparsifying transform domain.
3. the image is reconstructed with an optimization method that enforces both sparsity of the image representation and consistency of the reconstruction with the acquired samples.

The first condition ensures the image f is s-sparse. MR images meet that condition. The second condition ensures the position of the sampled coefficients is casual, i.e. they are collected without a specific, deterministic, pattern: this ensures the artefacts due to sampling are in the form of uncorrelated noise. MR acquisition can be designed to achieve incoherent undersampling. The third condition depends just on the reconstruction method. More details on CS method applied to MRI can be found in [22,23]. Though very important, CS can be efficiently applied if the number of collected samples, m, is not too different from n. This is for two reasons. First, the sparsity of the image, s, is unknown a-priori. Second, most CS applications, especially within medical imaging, have centered on the L_1-minimization problem because the corresponding L_0-minimization problem is intractable. An interesting recent improvement of CS regards the proposal of an innovative Homotopic L_0-Minimization [24] in which the authors describe a method for reconstructing MR images at sampling rates even further below that which are achievable using L_1-based CS methods by directly attacking the ideal L_0-minimization problem. Moreover, a practical scheme is presented for addressing the L_0 quasi-norm based on homotopic approximation using a wide class of deformable sparse priors, and an efficient semi-implicit numerical scheme for computation is described. The authors demonstrate both the problem tractability and the goodness of their results, when compared to the classical L_1-based CS methods, in spite of a reduction of the used samples for reconstruction.

Knopp et al. [25] present interesting results about the iterative reconstruction from non-uniform k-space sampled data though data sparsity is not reflecting CS requirements. They discuss about the effectiveness of using not-uniform FFT to reconstruct images directly from radial or spiral directions, through a generalization of the gridding process [26]. Very accurate results are obtained by using an iterative method to estimate density compensation weights, taking the result of gridding as a starting point. The best gridding results are obtained using the more expensive Voronoi weights. However, substantial improvement of the reconstruction quality is achieved during a small number of iterations for all used trajectories and weights.

Block et al [27] afford the problem of severely undersampled radial data in an iterative way with the usage of a TV constraint for the final image to reduce the strike artefacts produced by radial undersampling: also in this case the sparse samples do not fill the CS constraints. The reconstruction is obtained as a non linear optimization problem, solved through the conjugate gradient method. First, a search direction is estimated and, second, a line search into that direction is performed until the minimum of the functional in this direction has been identified. The search direction is obtained by calculating the gradient of the actual image estimate. At every step of the algorithm, the image estimate is mapped to the frequency domain. It is then controlled how well the estimate fits the measured data by calculating the difference. If the estimate is good enough, then the residuum vector contains only small entries, otherwise it contains large entries. In this case, the algorithm needs to know how to modify the image estimate to improve the match of the samples in the frequency domain. This information is obtained by mapping the residuum back to the image space. The reconstruction of an undersampled radial image through optimization still leads to streaking artefacts: the procedure does not measure the accuracy of the estimate at any other position in *k*-space than at the positions of the measured *k*-space coefficients. Being an underdetermined problem, more than one solution exist. To overcome this limitation, a penalty function is introduced into the optimization problem, based on the total variation constraint. The basic assumption of this idea is that the object consists of areas with constant (or only mildly varying) intensity, which applies quite well to medical tomographic images. If the object is piecewise constant, then the best representation of all image estimates that match at the spoke positions should be given by the one with the lowest derivatives at all pixel positions, that is the one minimizing the total variation, represented as the summation of the modules of the image second order derivatives (as discussed by the authors, the choice for the best derivative order still remain a debated argumentation). The obtained results are quite good regarding artefacts reduction, but some residual blurring is present.

Between the sample-independent methods prior to CS, Placidi et al. [28] describe an algorithm which is effective in reducing truncation artefacts due to missing *k*-space directions in MRI. The algorithm works first by filling the incomplete matrix of coefficients with zeroes and then iteratively adjusting the missing coefficients through a Fourier Transform method [29]. Then, this set of coefficients is used as a basis for a super-resolution algorithm that estimates the missing coefficients by modeling data as a linear combination of increasing and decreasing exponential functions with the Prony’s method [30]. The Prony’s method consists on the interpolation of a given data set with a sum of exponential functions: the MRI signals can be well represented as a sum of exponential functions and the missing data can be extrapolated by this representation. The algorithm can be used both for Cartesian and for radial undersampling, but it requires some computational overhead. A simpler variation of this method is reported in [31] where a simple constraint for iterative reconstruction, capable to deal with any sparse acquisition method, is used. The suggested methodology is based on the attempt to fill in the missing complex *k*-space values iteratively, by using the assumption that the image has to be zero outside a compact support. This approach transforms the original problem into an interpolation problem in the complex domain. The novelty is that it deals with iterative interpolation in the *k*-space based on the elimination of the artefacts from an extended support of the reconstructed image. Residual artefacts are reduced by using the method reported in [32]. The results, simulating different sparse acquisition strategies (Cartesian, radial, and spiral sampling), are not significantly different from those obtained by the original iterative method [28], though with very low computational overhead.

3 SAMPLE-DEPENDENT SPARSE METHODS

Different approaches are based on driving the acquisition process to adapt the collected signals to the sample shape.

Placidi et al. [33-35] present adaptive acquisition techniques for radial sampling MRI, first defined in the image space [33] and then in the *k*-space [34,35], to reduce the total acquisition time by collecting just the “most informative” trajectories, without any a-priori information on the image, but using information regarding the structured shape of the image collected during acquisition (in MRI, acquisition is a sequential process). This is possible through the calculation of a function, called entropy for its resemblance with the thermodynamic entropy, which measures the information content of each trajectory during the acquisition process, useful to discover sample internal symmetries, smooth or regular shape. In the *k*-space method [35] the entropy function is defined on the power spectrum of the projections. The process starts by measuring four regular orientations: 0°, 45°, 90°, and 135°. Then the evaluation of their information content is performed, followed by the selection of new angles where the information content is maximum. The next trajectory is measured between the two where the entropy function has a maximum. The procedure is repeated until the difference in entropy is significant. The method makes it possible to reduce the total acquisition time, with little degradation of the reconstructed image, adapting itself to the arbitrary shape of the sample (being able to catch eventual internal symmetries, low dynamic range and regular shape). The choice of, approximately, the most informative trajectories is made during the acquisition process, taking into account the information content of the previously collected data. The method allows the acquisition of a near optimal dataset, but not the optimal one. In fact, though very effective in reducing the acquisition time and undersampling artefacts, this method suffers from the following limitations: some important trajectories can be excluded from the acquired set, especially in the proximity of entropy function minima or maxima; some redundant trajectories can be collected, especially in the proximity of entropy function sharp variations. An effective application of the previous adaptive acquisition method has been also presented as a medical image compression strategy [36].

The method described in [37] considers the problem of measuring exactly the most informative set of radial directions by collecting a-priori information about the sample through the preliminary measurement of two circular paths at different distances from the *k*-space center. The idea behind the algorithm is that the power spectrum of a standard MR image is mainly distributed along specific *k*-space radial directions. These directions often terminate before the *k*-space border has been reached. Some of them do not start from the *k*-space center and extend to the *k*-space border. For taking into account these opposite situations, a set of preliminary circular trajectories are collected. Circular trajectories allow the interception of the most important radial trajectories. By analyzing the collected data, it is possible to establish the best set of trajectories before the image acquisition starts. For this reason, the acquisition process consists of the preliminarily collection of two concentric circular trajectories having the center in the image *k*-space center. The directions of the most informative trajectories can then be set by using the information acquired from the power spectra of these paths of coefficients. The set of these angular directions is optimal, most informative, set of trajectories to be collected in a standard way. Thought they require some preliminary time to collect the necessary information about the optimal angle set before the acquisition of radial projections started and, in some case, specialized hardware, the adaptive algorithms allow both the improvement in image quality and the reduction of the number of *k*-space coefficients with respect to other, non-adaptive, methods. Near optimal acquisition parameters are priory studied and set by using a numerical MRI simulation algorithm [38]. The adaptive methods proposed in [33-37] use a restoration/reconstruction method based on FFT and nearest neighbor interpolation [39]. Nearest neighbor interpolation is justified by the fact that close measured projections are very similar because of the used adaptive acquisition methods.

More recently, modifications to pure Compressed Sensing strategy, with the inclusion of selective sampling strategies, have been published. One of these, by Haupt et al. [40], propose a selective sampling procedure, called distilled sensing (DS) for the association with the purification occurring during the process of distillation, which is demonstrated to be effective for recovering sparse signals, supposed composed of non-negative values, in noise. DS is a sequential method which uses a collection of observations of the components of a sparse vector to identify and eliminate progressively the set of null components (absence of useful signal) while retaining the significant components. The process is a refinement of the observations which iteratively allocates more sensing resources to locations that are most promising while ignoring those that are unlikely to contain significant signal components. The method uses the fact that it is highly improbable that the signal (which is assumed to be positive) is present at locations where the observation is negative. The algorithm terminates after the final observation and its output is composed both by the final observations and by the set of locations measured in the last step. The DS application allows a significant improvement in effective signal to noise ratio (SNR) [41] compared to traditional compressed sensing without any adaptive selection method, though it is useful just for recovering positive signals.

An interesting paper [42] suggests and demonstrates that, by using an adaptive recovery strategy of wavelets transform coefficients of a signal by using their statistical dependences modeled using Hidden Markov Trees [43], the exploitation of the structure (the sparsity pattern) of structured sparse signals helps in improving results both with respect to traditional compressed sensing and with respect to adaptive sensing methods [41,42]. Moreover, its results fully support and justify the methods presented in [33-37] and discussed above.

Though the good experimental results obtained by adaptive strategies, in [18] the authors discussed the limits of any adaptive sensing technique with respect to the classical compressed acquisition/recovery scheme when it is necessary to afford the problem of acquiring arbitrary linear measurements F of a s-sparse n-dimensional vector f. In particular, they demonstrated that the following folk theorem is false: *the estimation error one can get by using an adaptive strategy which cleverly selects the next sampling trajectory based on what has been previously observed is far better than what is achievable by a nonadaptive strategy which sets sampling directions ahead of time, thus not trying to learn anything about the signal in between observations.* Though the previous folk theorem is false, it does not correspond to what a real case of adaptive sequential acquisition pursues. The following, applicable to sequential acquisition, version of the folk theorem is true: *The estimation error one can get by using an adaptive strategy which cleverly selects the next sampling trajectory based on what has been previously observed is better (in some cases, depending on the sample image, far better) than what is achievable by a nonadaptive strategy, tends faster to the minimal estimation error and furnish an accurate estimate of the sparsity value s of f (it allows a stopping criterion).*

For its demonstration, it is possible to use the arguments of the authors [18]. In particular, that an adaptive strategy could exist that gives an estimation error that is better than a non adaptive scheme has been demonstrated in [18]. In fact, the authors demonstrate that a clever adaptive strategy can reduce the number of collected measurements of at least $log(n/s)$ with respect to classical nonadaptive strategy with the same SNR, thus implying both that the clever strategy allows lower error and that it converges faster than a nonadaptive strategy.

The fact that an adaptive strategy could give an estimation error which is “far better” than a nonadaptive CS scheme depends on the underlying shape of the sample to be images. In fact, if the shape of the sample is regular, smooth and or symmetric, that is it has a structured shape, the adaptive scheme collects this information and terminates the acquisition process well before the nonadaptive CS scheme.

In fact, the classical blind method, having no information regarding f, has to fix m, the number of measurements, at a conservative value (at least $m=s\ log(n/s)$ [22,23], hopefully to about $m=3s$, but the value of s is unknown for classical CS!) in order to avoid artefacts in the form of high residual noise (that is, low signal to noise ratio). On the contrary an adaptive scheme, by collecting information of f during the acquisition both regarding the number of significant coefficients (in some space, it is capable of estimating s) and regarding their value, is able to stop the acquisition when the estimation error has reached a nearly constant value (the acquisition process, being incremental, allows the residual value to decrease almost monotonically until it has reached the experimental noise level). These are the reasons why adaptive schemes outperform the traditional CS strategy in practice.

The previous considerations fully justify the exploration of adaptive sparse sampling strategies and, more important, justify the proposal of adaptive compressed sensing strategies, as occurring in [44], where a Compressive Adaptive Sense and Search (CASS) algorithm is presented. CASS operates by dividing the signal into partitions and then using compressive measurements to test the presence of one or more significant (non-zero) elements in each partition. The procedure continues its search by bisecting the most promising partitions, with the goal of returning the largest s components (s is the sparsity of the signal, as above) of the vector. An analogous active search greedy algorithm has been proposed for robotic search from images [45] where it is also shown that the proposed adaptive technique can be substantially better than classical CS when the measurements are subject to the physically constraints of region sensing, especially if the physical space has low dimensions. In [46] the lower bounds performance regarding adaptive sensing for noisy sparse signal detection and support estimation are defined. Moreover, it defines the necessary conditions for the minimum amplitude for non-zero components and shows that the adaptive sensing strategies are essentially optimal and cannot be substantially improved.

Though interesting, the adaptive variations of CS reported above are not directly applicable to MRI. In fact: they are one-dimensional techniques and are incompatible with MRI acquisition requirements and trajectories (Figure 1); some of them require that the signal is non-negative and this is different from what occurs in MRI (oscillating signals of complex numbers). For the specificities of MRI acquisition and trajectories, slightly different adaptive techniques have been proposed.

The possibility of using a hybrid, adaptive-CS, sampling strategy for MRI is, for the first time, proposed in [47], where it has been verified that the adaptively, radially collected, samples verify the CS constraints and a L_1-norm based non-linear reconstruction can be used to obtain very accurate image reconstruction. After that, an alternative adaptive-CS method is presented that combines random sampling of Cartesian trajectories with an adaptive 2D acquisition of radial projections [48,49]. It is based on the evaluation of the information content of a small percentage of the *k*-space data situated in the central region of the sampling space, collected randomly but along Cartesian directions, to identify radial blades of *k*-space coefficients having maximum information content. The information content of each direction is evaluated by calculating an entropy function defined on the power spectrum of the projections. Besides that, the images are obtained by using a non linear reconstruction strategy, based on the homotopic L_0-norm [24], on the sparse data. The method overcomes classical weighted CS in image quality, though a lower number of collected samples is used. The use of homotopic L_0-norm minimization makes possible to obtain better reconstructions than by simple L_1-norm. The same method is also applied on cardiac MRI data in [50] with good experimental results. The previous method, though effective in reducing acquisition time while obtaining high quality images, is not optimal in the termination criterion (the estimation of the image sparsity), because some directions are wasted both for the definition of the first Cartesian dataset and for the blades measurement, and requires highly specialized software (specific MRI hardware is also recommended for allowing both single direction acquisition and blades of contiguous, parallel, directions). A simple, but efficient, iterative adaptive acquisition method (AAM) for radial sampling/reconstruction MRI is presented in [51]. AAM studies the inherent sparse structured pattern of the underlying image by analyzing the data acquired during the sequential acquisition to obtain useful information regarding the following "most informative" directions to be explored: the process is adaptive in the sense that it adapts the following directions to the shape of the object under investigation. The information regarding the shape of the object are collected through the wavelet analysis of a subsampled, approximation, image reconstructed by an initial subset of directions. The use of the wavelet domain is justified by its sparse nature and by its good properties of multiresolution and locality; the relationship between the MRI measurements in the *k*-space and wavelet domain is used to define the reconstruction process by finding the image f such that it is sparse in the wavelet domain and that its representation in the *k*-space is very close to the measured data.

AAM starts by considering a subset of equally spaced radial directions: for the given dataset, a reduced square support in the Fourier space is defined in which the resulting image is fully sampled. (*) Then, for this support the image is reconstructed. If the maximum image support is reached, the process is terminated and the reconstructed image is the final image. Otherwise, the wavelet transform of the image is calculated and the horizontal, vertical and diagonal details are estimated: the image is up-sampled of a factor 2 in the wavelet domain. Then, the inverse wavelet transform is calculated and finally the inverse Fourier transform of the resulting image is performed. In this case, a Fourier support up-sampled of a factor 2 is obtained for the unknown image. The resulting support is used for calculating the information content of the resulting coefficients and for calculating the position of "significantly informative" following directions (if any: in case of no further presence of significant directions, the acquisition process terminates). The new directions are measured, data integrated in the previous dataset and the process repeated with the image reconstruction (step * above). The up-sampling process in the wavelet domain is performed by using the following assumption [43]: if relevant information are present on the root, it is highly probable that relevant information are present also at the details levels and, in negative form, if relevant information are absent on the root, it is highly probable that they are also absent at the details levels. The distribution of relevant/irrelevant information to finer details is performed by using a *quadtree* representation: by knowing the position of an irrelevant (respectively, relevant) wavelet coefficient at a root of a *quadtree*, it is possible to assume information regarding the irrelevance of all its descendants in the tree (respectively, it is possible to estimate the details at finer scales, that is to interpolate, by using the correlation between the new root and its childs). The results, also in this case presented for cardiac MRI, demonstrate the advantages of an adaptive sensing strategy with respect to a classical CS scheme: an adaptive method tends to converge faster; it allows an acceptable estimation of the sparsity s of f, that is the adaptive method allows a strong reduction of the data (with respect to CS) necessary to reconstruct f with the same estimation error obtained by CS. Work is currently in progress in order to verify how to apply the adaptive strategy also to other MRI sampling strategies (for example, pure Cartesian, or spiral).

4 CONCLUSIONS

Recently MRI has developed considerably into the directions of dynamic imaging and fMRI opening up several new fields of application, in particular referring to real-time imaging. Although these developments are generally promising their application can be limited by the compromise between temporal and spatial resolution. To improve temporal resolution, undersampling is used.

A review of the most effective methods for sparse sampling acquisition and reconstruction, with the aim of reducing the undersampling artefacts, has been presented. In particular, methods have been classified in two classes: those methods that use sampling strategies whose patterns are independent of the sample shape, and those methods that adapt their action by modifying the sampling trajectories during the acquisition, i.e. the chosen trajectories (both in number and directions) are dependent on the sample shape. To the first class of methods allows also the traditional CS strategy, a very breakthrough technique which, for some extends, appears as counterintuitive: images can be almost exactly reconstructed by a sparse set of its Fourier coefficients, below the Nyquist requirements, if they are compressible and the collected coefficients are casually collected in the *k*-space. The second class contains some adaptive methods: they allow a strong reduction of the collected data if the *k*-space paths are collected through the most informative directions (most informative coefficients). Both CS and Adaptive methods require the images to be compressible, but they are completely different regarding the acquisition directions: in the first data are collected casually, in the second data are collected along the "most informative" trajectories. Though the usefulness of the adaptive strategies has been recently questioned [18], they overcome traditional CS in terms of acquisition time reduction, SNR increment and undersampling artefacts reduction. However, best results are obtained when hybrid adaptive CS strategies are used. For this reason, very promising hybrid strategies, are currently under investigation to obtain near optimal image reconstruction (as ensured by CS) with a reduced number of collected samples and SNR maximization (as ensured by an adaptive sampling strategy).

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