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3D GEOMETRICAL MATHEMATICAL STUDY AND VISUALISATION OF HUMAN UPPER LIMB MASS MOMENTS OF INERTIA

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Abstract: *Two joints upper human limb manipulator is designed that consist of three segments – upper and lower arm and a hand. By using a 3D geometrical mathematical model of it and its computer realization, the model provides data for upper limb's mass, volume, surface area, centre of mass coordinates, principal axes of inertia and principal moments of inertia, as well as moments of inertia taken at the origin of the laboratory coordinate system. We allow for a movement of the upper limb parametrized by two angles – one that determines the position of the upper arm that moves in the plane of the body while the second determines the position of the lower arm with respect to the upper one. We determine and present data how the centre of mass of the upper limb changes when it moves, so that its end reaches, say, the area of the mouth of the human. In addition, we present corresponding data for the changes of the moment of inertia along such movement. The presented study can be helpful in the design of manipulation systems with application in rehabilitation. More generally, the method provides a handy opportunity to calculate the mass-inertial parameters of the upper limb of a given individual, provided the anthropometric easily measurable data for that individual are known.*

1 INTRODUCTION

A biomechanical geometrical model of the human upper limb is presented, with the help of three-dimensional (3D) geometrical mathematical modeling. This model represents a modification of the ones suggested in [1, 2] and is partially based on our own anthropometric measurements of 50 men aged between 30–40 years. The anthropometric data needed for the generation of the model were reported in [3, 4, 5].

The investigation on the human body geometric and mass-inertial characteristics is of key importance in human motion analysis. The current investigation is a natural continuation of the work, presented in [6, 7]. The objective of the work presented in this paper is a definition of the mass properties of a two joints human upper limb manipulator. In order to illustrate the main features and advantages of the proposed design concept, examples of two different positions of the upper limb are presented in Figs 1 and 2. We define a coordinate frame $\boldsymbol{F}$ with its origin at the midpoint of the shoulder joint with the z-axis along the body vertically downwards, the x-axis in the sagittal plane, and the y-axis is in the transversal plane. For any of the two positions we present the coordinates of the centre of mass, the principal moments of inertia, the components of the unit vectors that define the coordinate frame in which the inertial tensor is diagonal, and the moments of inertia with respect to $\boldsymbol{F}$, as well as its components with respect to a system that has its centre at the centre of mass but with axes aligned with $\boldsymbol{F}$. Next, we present data how the centre of mass of the upper limb changes when the limb moves so that its end reaches the area of the mouth of the human – one imagines a movement of a disabled person who shall be helped in order to drink a cap of water, eat a meal, etc. Visualisation of the motion of manipulator's mass centre is performed under the variation of both investigated angles, generating a saddle-like surface in the original laboratory coordinate system. Finally, for any of these centres of mass the conjugated mass moments of inertia are visualized by generating and graphically presenting the corresponding ellipsoids of mass moments of inertia. In order to achieve the above, we have performed the following main tasks:

1) Building up a 3D geometrical mathematical model of the upper human limb manipulator, using the statistical data presented in [3, 4, 5];
2) Verification of the proposed model via comparison of the data obtained from the determination of the human upper limb manipulator mass properties by using the computer code and those derived by analytical means from the previous studies. The comparison demonstrated a nice agreement between these data.
3) Investigation on how the centre of mass of the proposed manipulator changes with respect to angle variations.
4) Visualization of the mass centre motion in 3D space.
5) Visualization of the mass moments of inertia on the basis of the movements of the mass centre in 3D space.
6) Derivation of data that can be used in the design of devices aimed to help disabled people, having problems with the movement of their upper limbs.

We consider a model in which the upper limb is characterize with three segments – upper arm, lower arm and a hand, and has degrees of freedom due to two joints positioned at the shoulder and in the elbow. We take that the upper arm can move only in the (z, x) plane with an angle α that characterizes its position. The lower arm is taken to be able to move in the plane formed by the upper and the lower arm with angle β determining the position of one with respect to the other one. The proposed topic seems to be very interesting and attracting the attention of many scientists worldwide. For example, investigations on a human-arm-like mechanical manipulator are reported before in Refs. [6, 7, 8, 9, 10, 11, 12]. Reference [9] can be used as a general source of knowledge on the subject. In addition, upper limb portable motion analysis system, based on inertial technology, is revealed in [13] for neuro-rehabilitation purposes. Reference [14] deals with control of hand trajectories in multi-joint arm movements. In [15] one investigates the coordination of arm movements and present there a mathematical model that has been experimentally confirmed. The effect of visuomotor displacements on arm movement paths is presented in [16]. Finally, the mechanics of multi-joint posture and movement control are revealed in Ref. [17]. The model mass properties and the corresponding mass moments of inertia for females [7] can be also computed, applying the corresponding material densities for women and the geometrical characteristics pertinent to them for, otherwise, the same model geometry. In [7] the calculated results for the three human upper limb parts for females are compared with the analytically already calculated and reported in [6] mass moments of inertia for males. The agreement between the computer model and the analytical results calculated in few basic positions for any of the segments gives us confidence, that the model can be used for determining the mass inertial characteristic of the whole upper limb in any of its admissible spatial positions.

2 BUILDING UP OF THE MODEL

The ideology of the analysis via 3D geometrical mathematical modelling is *i)* creation of suitable geometrical model of the investigated body, and *ii)* by applying of the geometry, the material density and the analytical properties of the 3D bodies, used in the approximation, to determine all properties of interest including the body mass properties and the mass moments of inertia, respectively [6, 7]. The model geometry of human upper limb used in the current study for males is presented in Figs 1 and 2. We recall that the upper arm extends between *acromion-radiale* anthropometric points, while the lower arm is taken to be, as usual, between the *radiale-stylion* anthropometric points.

The role of the angles α and β and the principal positions of the upper limb in our model are depicted in Figs. 1 and 2. Figure 2 demonstrates the role of the angle β for the movement of the lower arm, when the upper arm is simply kept fixed aligned with the body along the z-axis. Axes 1-5 in Figs. 1 and 2 are automatically created by the computer system and represent three axes in the basic co-ordinate system X, Y, Z with origin in the shoulder and the other two are oriented longitudinally along the upper and the lower arm, respectively.

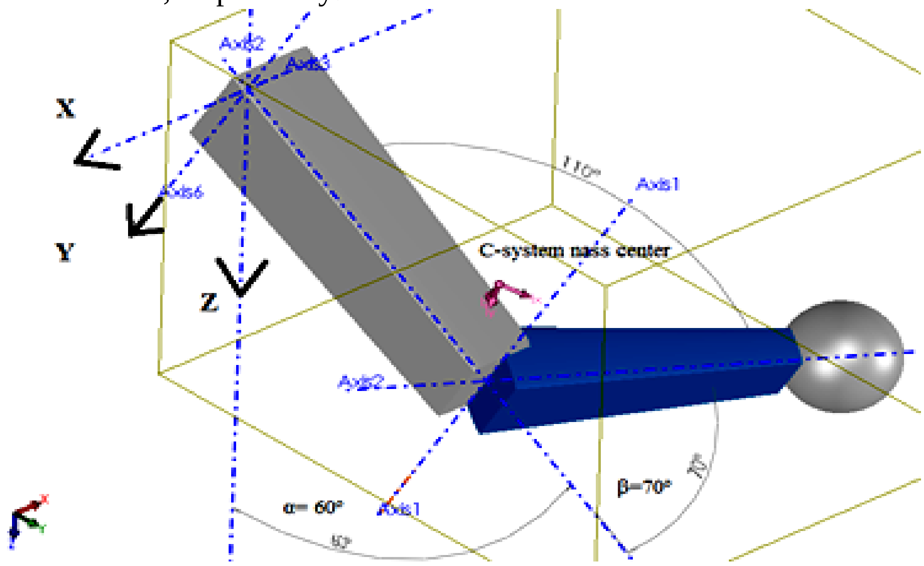

Figure 1: Manipulator with two joints, position α= 60º, β=70º.

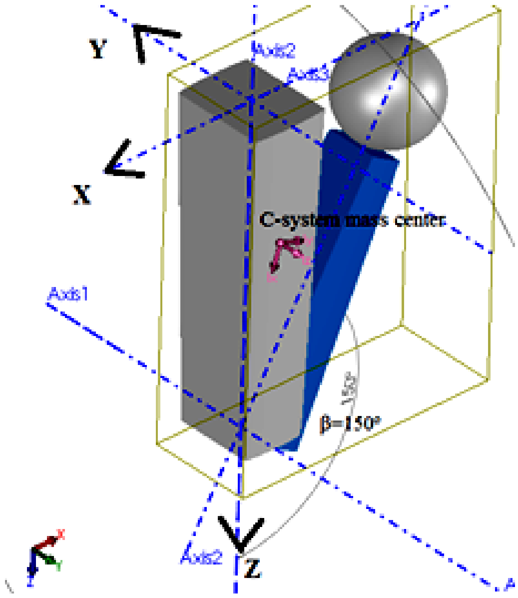

Figure 2: Manipulator with two joints, position - α= 0°, β=150°.

3 RESULTS FROM THE MODEL

Before reporting the mass inertial parameters for the upper limb in the above positions let us state that our computer model generated in CAD media SolidWorks, has been verified against analytical calculation of the mass inertial parameters for any of the components of the upper limb which includes the volume, mass, position of centre of mass and the inertial moments of the components [5]. That is why we have a confidence that it reliably works for the upper limb as a whole when one is interested in these characteristics in general positions of the upper limb. In principle, our computer model can provide data for mass, volume, surface area, centre of mass-coordinates, principal axes of inertia and principal moments of inertia, moments of inertia taken in a coordinate frame at the centre of mass and axes aligned with the output coordinate system and moments of inertia, taken at the output coordinate system. For example, this data for the upper limb position shown in Fig. 1 are:

Mass = 3248.13 g, Volume = 3015170.96 mm^3 , Surface area = 193039.49 mm^2, Centre of mass coordinates (in mm): $C_x = -67.97, C_y = -205.80, C_z = -205.80$. The point C represents the centre of mass of the manipulator in the configuration given. It also is the origin of the coordinate system shown in Figs 1 and 2, in which the inertial moment's tensor is in its diagonal form. In configuration of Fig. 1 the unit vectors defining the principal axes of inertia are with components $\vec{e}_1 = (0.59, 0.70, -0.40)$, $\vec{e}_2 = (-0.81, 0.51, -0.30)$, and $\vec{e}_3 = (0.00, 0.50, 0.87)$, and the principal moments of inertia: (in units of kg × dm^2) are

$$P_x = 1.25, P_y = 8.3, P_z = 9.2 \tag{1}$$

Moments of inertia taken at the centre of mass *C* and aligned with the output coordinate system are:

$$\begin{array}{lll} Lxx = 5.8 & Lxy = 2.9 & Lxz = -1.7 \\ Lyx = 2.9 & Lyy = 5.1 & Lyz = -2.4 \\ Lzx = -1.7 & Lzy = -2.4 & Lzz = 7.9 \end{array} \tag{2}$$

while moments of inertia taken at the output coordinate system are:

$$\begin{array}{lll} Ixx = 24.2 & Ixy = 7.5 & Ixz = -4.3 \\ Iyx = 7.5 & Iyy = 11.2 & Iyz = -10.3 \\ Izx = -4.3 & Izy = -10.3 & Izz = 23.1 \end{array} \tag{3}$$

Similar to what has been presented for the limb in position of Fig.1, detailed data can be reported for any other position of the upper limb. For example, for the configuration shown in Fig. 2, one obtains:

$$C_x = -36.16, C_y = 0, C_z = 150.26 \tag{4}$$

Using the possibilities offered by the model, we have determined how the centre of mass of the upper limb changes, when it moves in a way that the hand reaches near the mouth of the human. Figure 3 shows the C-mass centre-coordinates in [mm]: α varies with seven values from zero to 60º, while β varies with eleven values from 70º to 170º. A saddle like surface of the positions of the centre of mass is defined in this way.

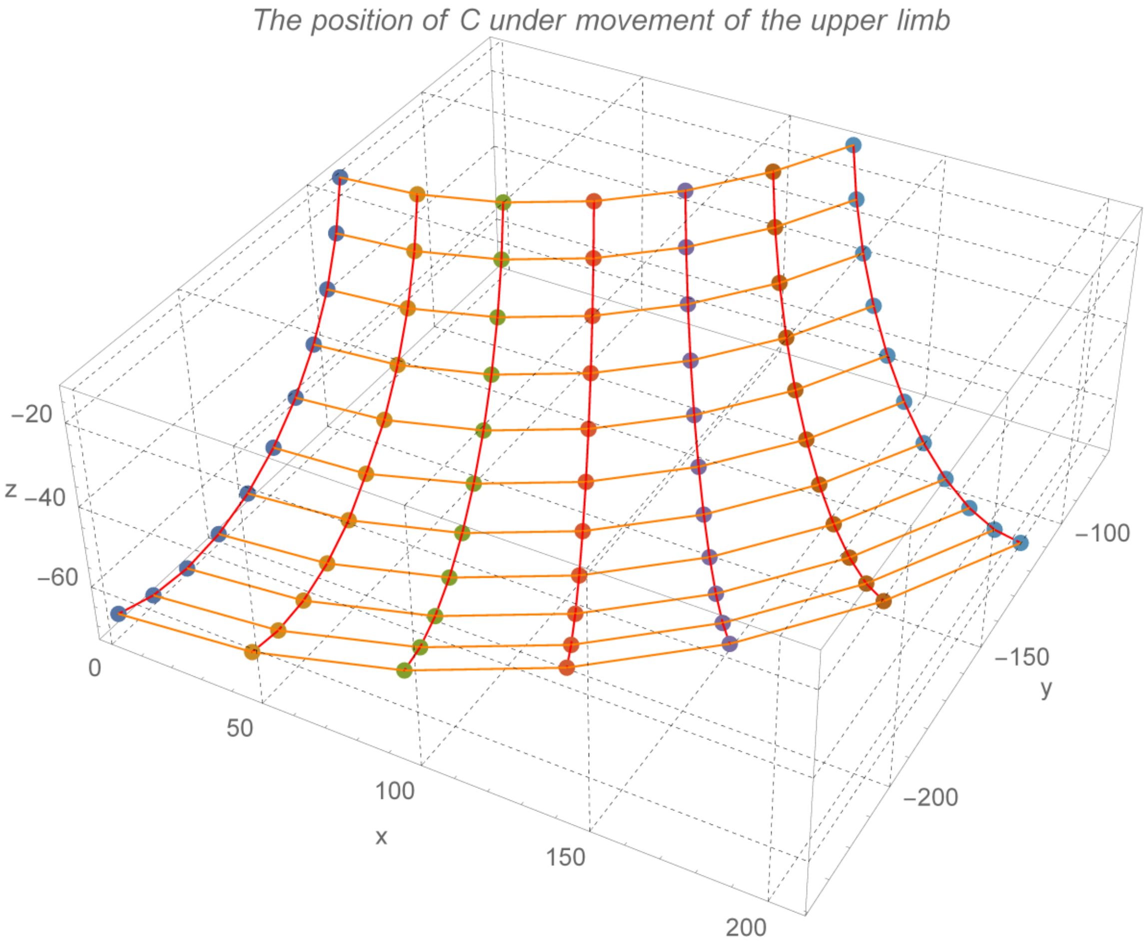

Figure 3: Graphics of mass centre of motion-coordinates in [mm].

4. 3D VISUALISATION AND COMPARISON

Visualisation of the motion of manipulator’s mass centre is performed under the variation of both investigated angles generating a particular saddle-like surface in the original laboratory coordinate system. For any of these centres of mass the conjugated mass moments of inertia are visualized by generating and graphically presenting the corresponding ellipsoids of mass moments of inertia.

Table 1 shows the C-mass centre-coordinates in [mm] and mass moments of inertia.

	C- Mass centre	coordinates [mm]	Mass moments of Taken at the output.	inertia [кg.cm^2]; coordinate system
Beta	Alfa=0^o	Alfa=60^o	Alfa=0^o	Alfa=60^o
70^o	X =- 67.97 Y = -0.00 Z = 237.63	X = -67.97 Y = -205.80 Z = 118.82	Ixx = 2418.96 Iyy = 2907.19 Izz = 523.40	Ixx = 2418.97 Iyy = 1119.35 Izz = 2311.25
80^o	X = -71.23 Y = -0.00 Z = 225.46	X = -71.23 Y = -195.25 Z = 112.73	Ixx = 2125.9 Iyy = 2662.7 Izz = 572.05	Ixx = 2125.87 Iyy = 1094.73 Izz = 2140.07
90^o	X = -72.33 Y = -0.00 Z = 212.90	X = -72.33 Y = -184.37 Z = 106.45	Ixx = 1856.85 Iyy= 2410.62 Pz = 588.94	Ixx = 1856.86 Iyy = 1044.37 Izz = 1955.21
100^o	X = -71.23 Y = -0.00 Z = 200.34	X = -71.23 Y = -173.50 Z = 100.17	Ixx = 1621.6 Iyy = 2158.5 Izz = 572.05	Ixx = 1621.64 Iyy = 968.66 Izz = 1761.90
110^o	X = -67.97 Y = -0.00 Z = 188.16	X = -67.97 Y = -162.95 Z = 94.08	Ixx = 1425.8 Iyy = 1914.06 Izz = 523.4	Ixx = 1425.84 Iyy = 871.07 Izz = 1566.40
120^o	X = -62.64 Y = -0.00 Z = 176.73	X = -62.64 Y = -153.05 Z = 88.37	Ixx = 1271.01 Iyy = 1684.7 Izz = 448.86	Ixx = 1271.01 Iyy = 757.83 Izz = 1375.74
130^o	X = -55.41 Y = -0.00 Z = 166.40	X = -55.41 Y = -144.11 Z = 83.20	Ixx = 1155.12 Iyy = 1477.38 Izz = 357.43	Ixx = 1155.14 Ixy = 242.65 Izz = 1197.40
140^o	X = -46.49 Y = -0.00 Z = 157.49	X = -46.49 Y = -136.39 Z = 78.74	Ixx = 1073.48 Iyy = 1298.44 Izz = 260.13	Ixx = 1073.48 Iyy = 519.71 Izz = 1038.86
150^o	X = -36.16 Y = -0.00 Z = 150.26	X = -36.16 Y = -130.13 Z = 75.13	Ixx = 1019.75 Iyy = 1153.28 Izz = 168.70	Ixx = 1019.76 Iyy = 414.85 Izz = 907.14
160^o	X = -24.74 Y = -0.00 Z = 144.93	X = -24.74 Y = -125.51 Z = 72.46	Ixx = 987.33 Iyy = 1046.32 Izz = 94.16	Ixx = 987.34 Iyy = 332.21 Izz = 808.29
170^o	X = -12.56 Y = -0.00 Z = 141.67	X = -12.56 Y = -122.69 Z = 70.83	Ixx = 970.48 Iyy = 980.82 Izz = 45.52	Ixx = 970.48 Iyy = 279.34 Izz = 746.99

Table 1: C-mass centre-coordinates in [mm] and mass moments of inertia [kg.cm^2], taken at the output co-ordinate system for α= 0^o and α=6 0^o, β=70^o - 170^o in an interval of 10 degrees for each α

The coordinates of *C* and the moments of inertia in units of [kg.cm^2] are with respect to the output laboratory co-ordinate system for α values between zero and 60°, while the angle β varies in this table with eleven values from 70° to 170° for each value of α. Mass moment of inertia ellipsoid for α= 0°, β=70° is visualised in Fig. 4 and mass moment of inertia ellipsoid for α= 60°, β=70° is visualised in Fig. 5. Data for the two mass moments of inertia are taken from Table 1. It is obvious, from Figs 4 and 5 the difference between the dimensions of both ellipsoids, due to the variation of both manipulator position's parameters.

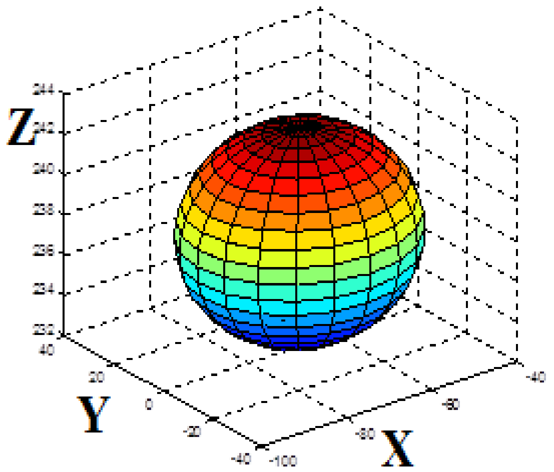

Figure 4: Mass moment of inertia ellipsoid visualisation for manipulator position α= 0°, β=70°.

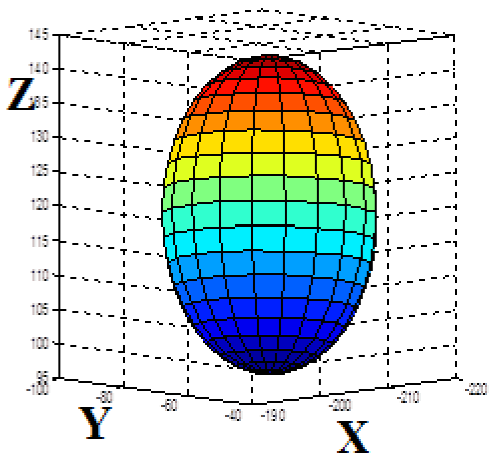

Figure 5: Mass moment of inertia ellipsoid visualisation for manipulator position α= 60°, β=70°.

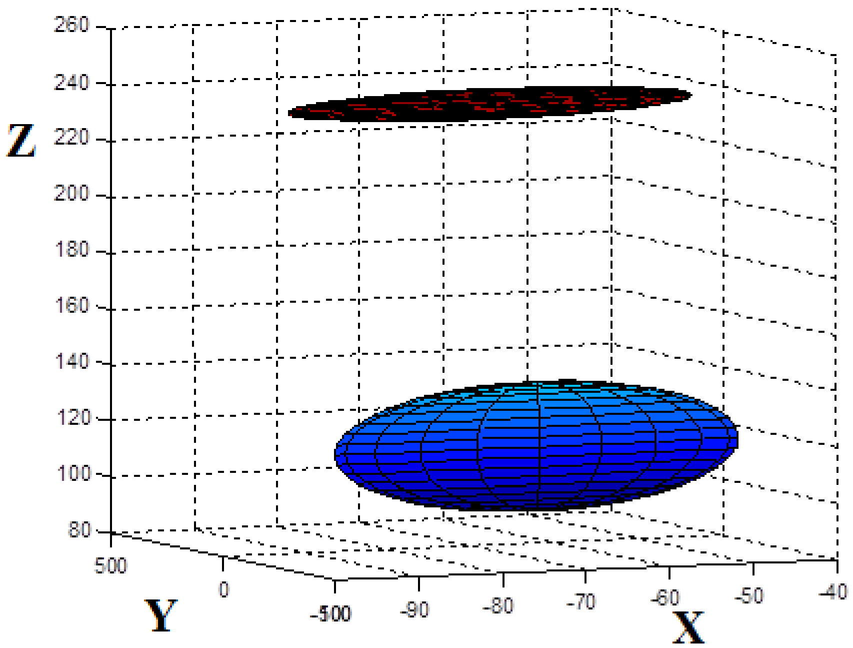

Figure 6: Mass moments of inertia shown in Figs 4 and 5 plotted together.

Figures 4 and 5 are on a different scale that is why the reader cannot see the difference between the two mass moments of inertia very well. In order to achieve that, we plot both mass moments of inertia simultaneously, we get the plot shown in Fig. 6.

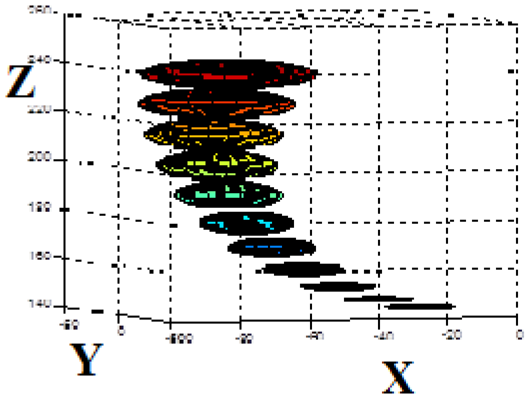

Figure 7: Mass moments of inertia ellipsoids visualisation for manipulator position α= 0°, β varies with eleven values from 70° to 170°.

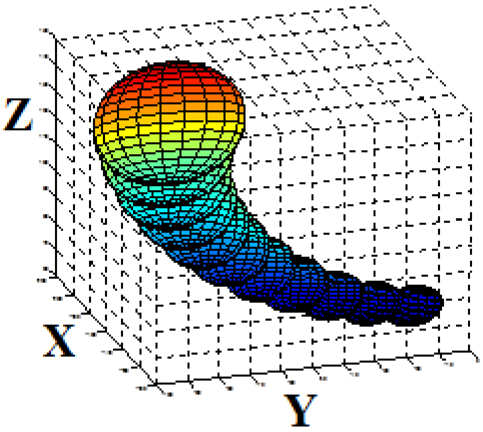

Figure 8: Mass moments of inertia ellipsoids visualisation for manipulator position α= 60°, while while β varies with eleven values from 70° to 170.

Figures 7 and 8 are aimed to show the difference between the inertial ellipsoids for two different values of α when β runs from 70° to 170°. For the whole set of parameters, consider when determining the position of the centre of mass, see Fig. 3, the corresponding ellipsoids of inertia are calculated in visualized in Fig. 9.

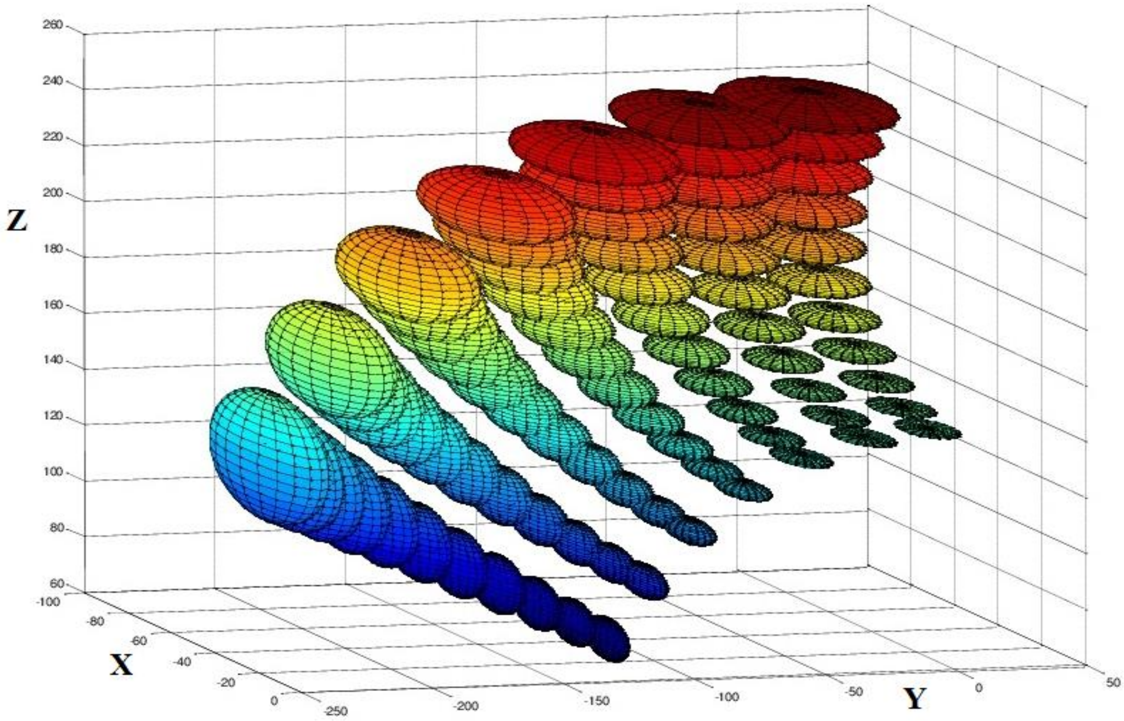

Figure 9: Mass moments of inertia ellipsoids visualisation for α varies with seven values from zero to 60°, while β varies with eleven values from 70° to 170° in an interval of 10°.

5 SUMMARY, CONCLUSIONS, AND FUTURE WORK

In this paper, a new approach for determination of the human upper limb mass-inertial characteristics is presented based on using the 3D geometrical mathematical model of the limb – see Figs 1 and 2, and the corresponding analytical properties of the bodies used to model the segments of the limb, as well as the advantages of computer realization of the model in CAD system Solid Works. We derived data how the centre of mass of the upper limb changes when the limb moves, so that its end reaches the area of the mouth of the human, see Table 1 and Fig. 3, as well as the moments of inertia – see Figs. 4-9 and Table 1. The data reflect how the corresponding characteristics change under a movement of a disabled person who shall be helped in order to drink a cap of water, eat a meal, etc.

Looking at the influence of both angles variation on the mass moments of inertia values, it is obvious that at fixed value of angle α when angle β varies from 70^o to 170^o all the mass moments of inertia Ixx, Iyy, Izz constantly diminish. This phenomenon can be clearly observed in Figs 7 and 8. The difference between the maximal and minimal values is about 60% in X direction, 65% in Y direction and 90% in Z direction. Looking at the influence of variation of angle α when it changes from 0^o to 60^o with the angle β fixed, one concludes that the mass moments of inertia Ixx, Iyy, Izz in X direction remain almost unchanged, while in Y direction they constantly diminish with about 60% among the maximal and minimal values; in Z direction the corresponding values constantly increase with about 80% between the maximal and minimal values. This tendency can be clearly observed in Fig. 9.

An obvious direction of future activities is the improvement of the proposed model so, that the geometrical modelling becomes as close as possible to the real shape of the human upper limb. In the current study, we used the geometrical parameters directly as measured from a set of individuals. The approximation with specific 3D bodies of the segments of the limb leads to a given mistake in reproducing its mass. If one insists on using the physically measured density, one way around that problem is to modify with, say, 5% precision the actual measured data in such a way, that the obtained mathematical model reproduces exactly the mass of segments of the limb as follows from the corresponding regression equations. Since they do depend on the gender of the corresponding person, our model will also become, to a given extend, gender specific.

Authors believe that the presented study can be really helpful in the design of manipulation systems with application in sport and rehabilitation, providing the handy opportunity to calculate and determine the mass-inertial parameters of the upper limb of a given individual provided the anthropometric easily measurable data for that individual are known.

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