

## PHYSICAL FOUNDATIONS FOR THE SELECTION OF DIAGNOSTIC PARAMETERS OF ATHEROSCLEROTIC PLAQUE GROWTH

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**Abstract:** *The paper presents derivation of a diagnostic parameter that may become the basis for estimation of the level of blocking the carotid artery lumen with plaques. The mathematical analysis showed that with a 60% increment in the atherosclerotic lesions, the diagnostic parameter changes rapidly – above that value total blockage of the arterial lumen might occur. 4 cases have been analyzed: circular, laminar, lenticular and eccentric growth of plaques. It appeared that the determined parameter is independent of the shape or geometry of the depositing plaques. Such independence has been obtained through reducing the variable radius of artery with plaques to hydraulic radius. The results obtained through analysis were compared to numerical studies. The purpose of the simulations was to find out the impact of the deposit thickness on significant reduction of the flow through the channel and whether the value determined following the numerical studies will converge with that of the analytical studies. The growths were modeled as a turbulence occurring on one side of the channel or as a channel with bilateral turbulence. The analysis of the simulation results has shown that with a ca. 60% stenosis, the energy of turbulence grew rapidly and the whirlpools that occurred were sufficiently large to cause destabilization of the flow. The studies carried out represent analytical and numerical evidence that if a patient's deposits on the arterial wall exceed 60% of the arterial wall lumen, such patient will be explicitly qualified for operation of the plaque removal, because any further growth of the plaques is very rapid and may cause blood flow blockage.*

## 1 INTRODUCTION

There are numerous causes of arterial stenosis or occlusion. Incorrect blood composition causes deposition of solid particles, e.g. fat plaques deposit on arterial walls causing a significant reduction of the arterial flow lumen. Arteries with relatively large diameters are usually exposed to stenosis. Plaque deposition on the walls is often caused by atherosclerosis [1,2]. The disease is of a chronic and degenerative nature, because the atherosclerotic plaques and thromboses increase along with the progress of the disease. Based on the experiments [3,4], it was proved that 75% stenosis of the flow lumen causes a significant drop in the blood tension. The studies contained in this paper were aimed at determining of a parameter that would explicitly determine the value of the progressive stenosis. The parameter was validated through numerical examination of the flow phenomena taking place after the narrowing for different stenosis values.

The plaques are most frequently deposited in four basic forms: laminar, circular, eccentric and lenticular. Figure 1 presents the graphic interpretation of these conditions. Whatever the nature of the plaques, we may assume that they develop in even volumes resulting from the life style. Thus, all the four cases can be brought to one – circular development of plaques. This is caused by maintaining the same drop in the blood pressure in non-circular and circular lumen.

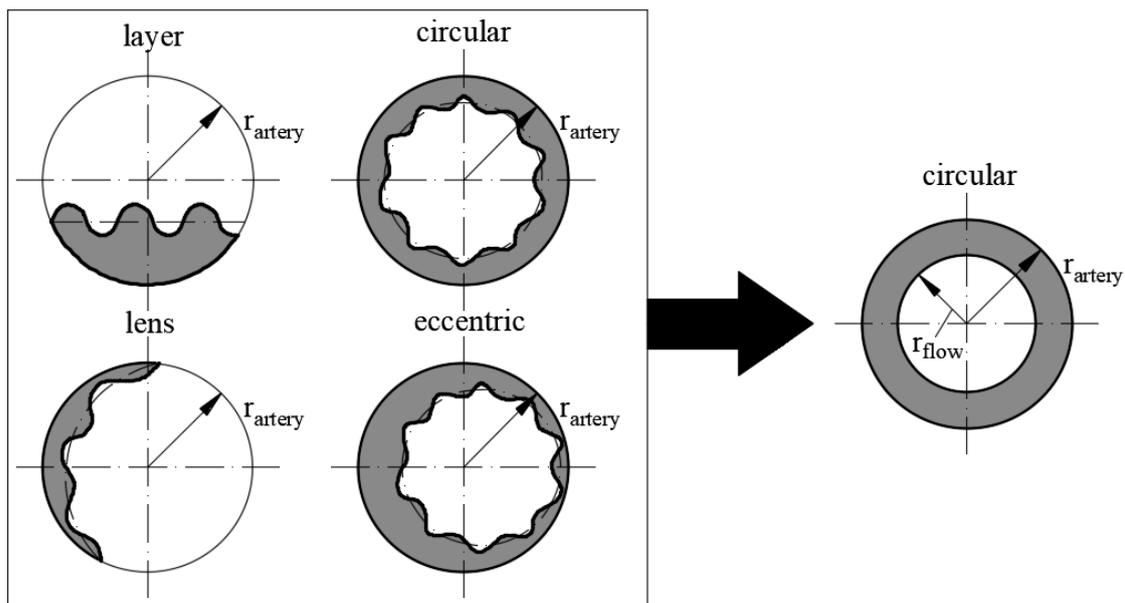


Figure 1: Characteristic plaque deposition: a) layer, b) circular, c) lens, d) eccentric.

The diagnostic parameter was defined as a function of plaque field growth (or reduction of the hydraulic radius) to the growth of the surface of the depositing plaques. The characteristics carried out show that beginning with 60% arterial lumen occlusion, the value of the growth of the diagnostic parameter becomes very rapid along with the growth of the plaque deposition, while the arterial flow radius decreases. The curves were compared to the numerical analysis where the results converged with what had been proven analytically. The final results are the foundation for qualifying the patients for surgical removal of arterial lesions.

## 2 ANALYTICAL METHODS

As has already been mentioned, this paper [1] includes a consideration of a change of the hydraulic radius of the plaques as a function of growth of the plaque field. The possible ways of plaque deposition presented in Fig. 2 can be mapped on a circular flow, maintaining the same drop of pressure. It enables a derivation of other parameters characterizing plaque deposition. This process can be described geometrically in the following way: secretion of the plaque volume  $V_z$  by the system depositing in the form of a stripe of thickness  $\delta$ , width  $s$  and length  $l$ , as presented in Fig. 2

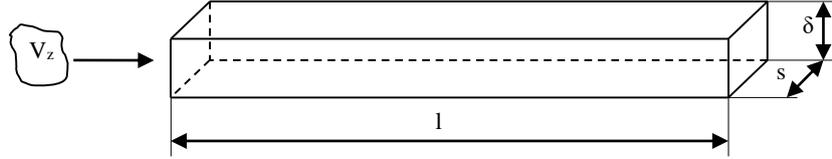


Fig. 2. The concept of transformation of the plaque volume  $V_z$  into a stripe

The length of the stripe at the first stage of plaque deposition in the artery of radius  $r_0$  is  $l = 2\pi r_0$ , Fig. 2. The thickness of the stripe (very small volumes  $V_z$  considered) is the result of the plaque's volume.

$$V_z = 2\pi r \cdot \delta \cdot s$$

therefore, the further increases of the stripe thickness in the form of a cuboid with decreasing height  $l$ , are, Fig. 3

$$\delta_1 = \frac{V_z}{2\pi r_0} \cdot s$$

$$\delta_2 = \frac{V_z}{2\pi r_1 \cdot s} = \frac{V_z}{2\pi s(r_0 - \delta_0)}$$

$$\delta_3 = \frac{V_z}{2\pi r_2 \cdot s} = \frac{V_z}{2\pi s(r_0 - \delta_1 - \delta_2)}$$

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$$\delta_{i+1} = \frac{V_z}{2\pi r_i \cdot s} = \frac{V_z}{2\pi s(r_0 - \delta_1 - \delta_2 - \dots - \delta_i)}$$

The flow area  $A_{\text{flow}} = \pi r_i^2$  and the plaque area  $A_{\text{plaque}} = \pi r_0^2 - A_{\text{flow}} = \pi(r_0^2 - r_i^2)$

or

$$\delta_{i+1}^r = \frac{1}{2} \cdot \frac{V_z}{\pi r_0^2 s} \cdot \frac{1}{1 - \delta_1^* - \delta_2^* - \delta_3^* - \dots - \delta_i^*}, \quad \delta_i^* = \frac{\delta_i}{r_0}$$

$$r_i = r_0 - \sum_{j=1}^i \delta_j$$

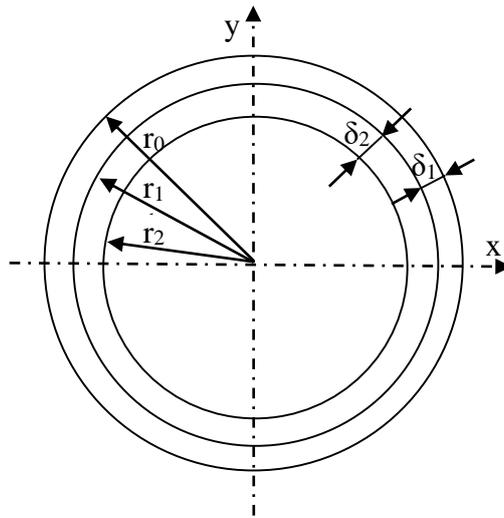


Fig. 3. Concentric growth of plaques

Fig. 4 presents the curve of the diagnostic parameter as a function of relative plaque cross-section (related to the arterial surface). The function values were determined for the arterial radius of 5 mm. Fig. 4 also shows the cross-section of such an artery with a subsequent decrease of the arterial lumen radius by 1 mm.

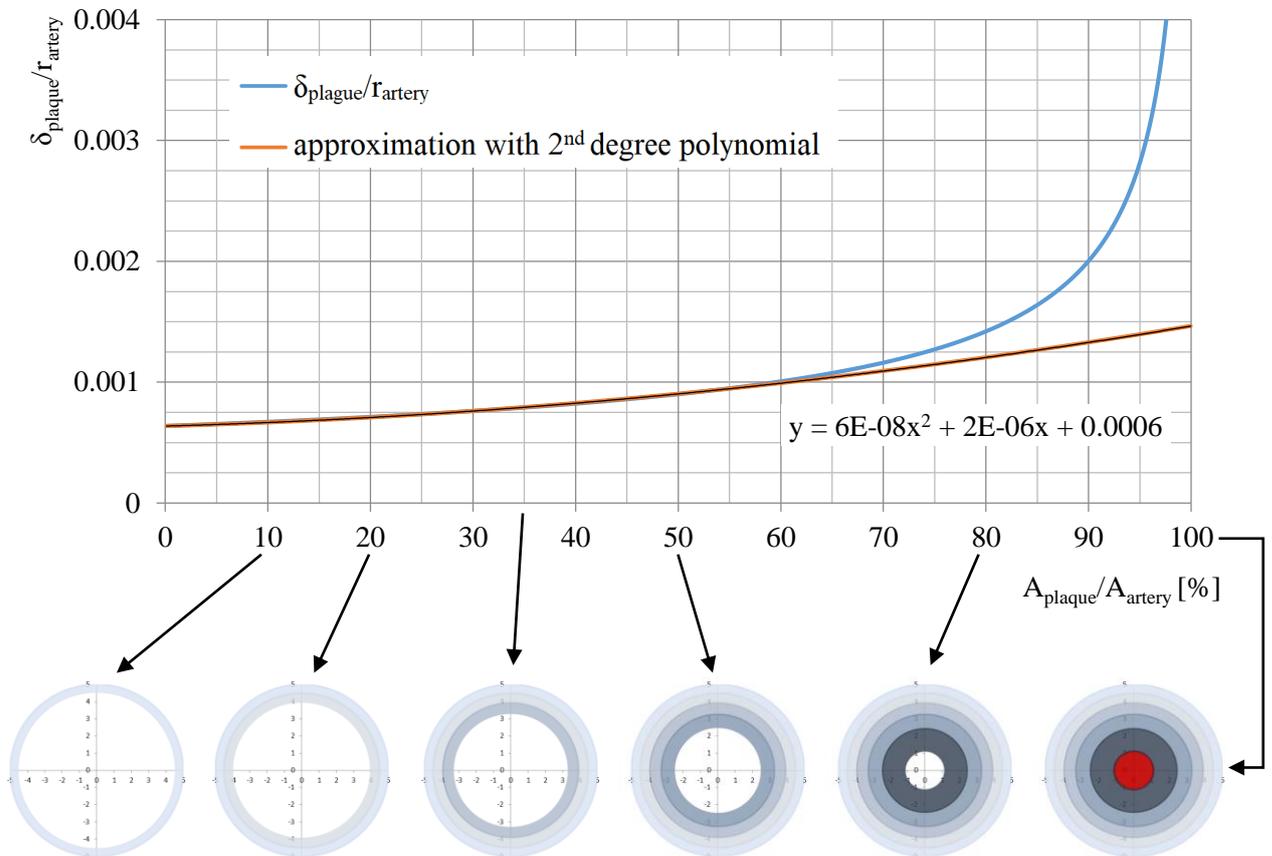


Figure 4: Subsequent increases in plaque layers  $\delta_{\text{plaque}} / r_{\text{artery}} = f(A_{\text{plaque}} / A_{\text{artery}})$

The determined curve was approximated by a second-degree polynomial. We can observe a significant deviation of the curve of the original function. Up to 60%, the curves converge, however, the differences from the original function begin to grow very rapidly above the said value. Upon exceeding 60%, it grows exponentially to infinity. The graphical representation also indicates that the arterial occlusion level is observable, while the subsequent growth of the plaque width leads to an almost total reduction of the arterial lumen.

The same trend of dramatic growth above 60% is also shown by the function presented in Fig. 5. It presents the rate of growth of the plaque thickness. As the subsequent numerical studies have shown, the growth of the said rate is determined by the growth of the velocity of the vortices downstream of the stenosis.

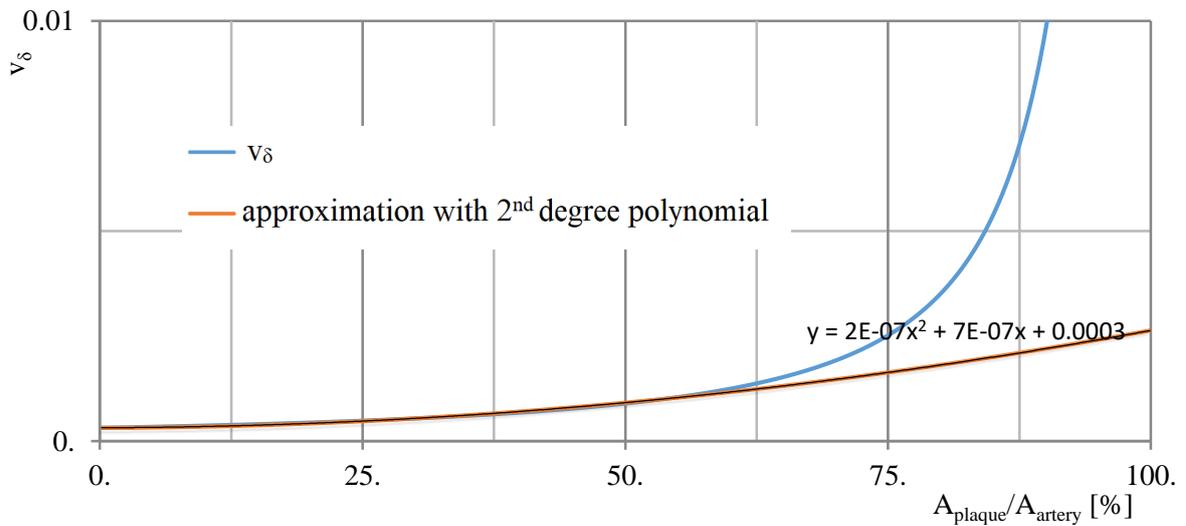


Figure 5: Plaque growth rate  $v_{\delta} = f(A_{\text{plaque}} / A_{\text{artery}})$

Fig. 6 presents the function of a drop of the radius of the flow lumen - along with the growth of the plaque area the flow radius decreases linearly, while above 60%, a significant deviation from the approximated curve occurs, function I decreases exponentially to infinity.

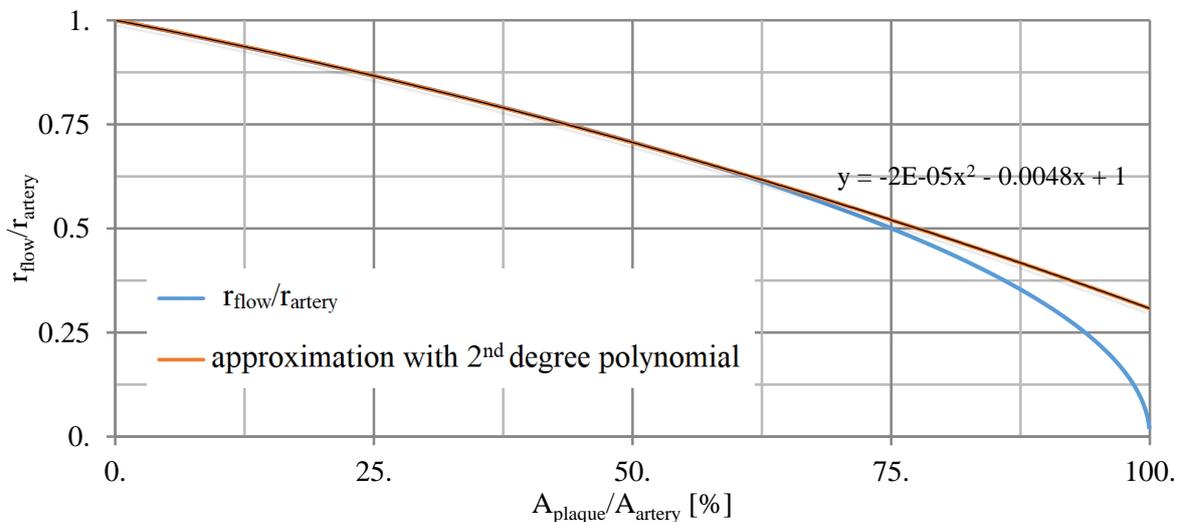


Figure 6: Decrease in the radius of the flow cross-section  $r_{\text{flow}}/r_{\text{artery}} = f(A_{\text{plaque}} / A_{\text{artery}})$

### 3 NUMERICAL RESULTS

Simulations of blood flow through the artery were made with the use of Fluent software in order to present the flow field after the critical flow lumen value had been reached. The depositing plaques were modeled as a bell-shaped obstacle in a straight-line canal. A non-stationary, pulsating velocity profile with a maximum value of 0.4 m/s was introduced in order to map the blood flow nature as accurately as possible. Blood was modeled as a pseudoplastic non-Newtonian fluid defined by the Carreau function [5,6]. Fig. 7 presents the curve of the current line: 60%, 65%, 70% and 75%.

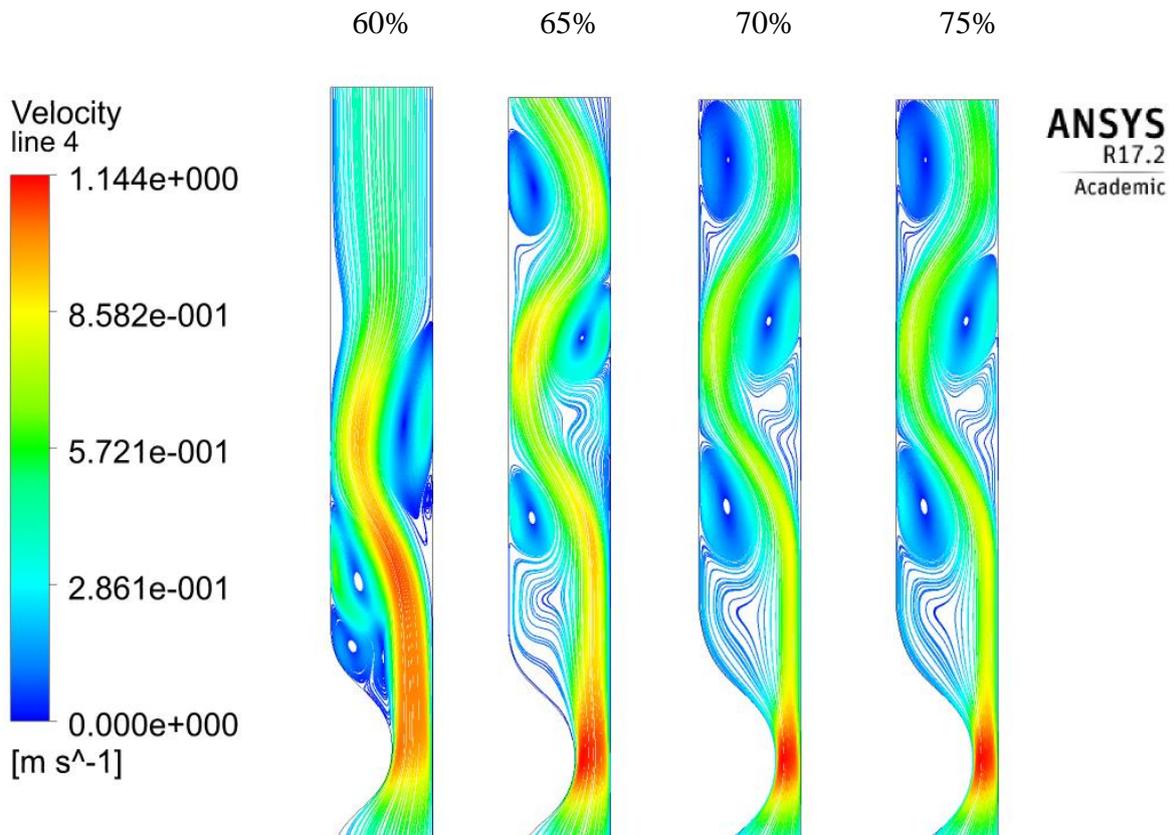


Figure 7: Numerical results

Vortexes are generated downstream of the stenosis causing deposition of further plaques on the walls. The area and velocity of the vortexes grow along with the increase in the stenosis, which causes the increase in the plaque deposition. The phenomenon of vortex multiplication occurs. In addition, they are generated interchangeably on both sides of the walls causing that the flow lumen is not of a straight-line nature. The blood must flow through a curved track in order to pass the disturbed flow area, which causes additional distortions of an already unstable flow.

## 4 CONCLUSIONS

Whatever the nature of the plaque deposition, thanks to the application of the hydraulic radius (mapping of irregular fluid flow cross-section), all the cases can be brought to a simple case of a regular, circular growth of plaques. Such a simplification can be used owing to the fact that the overall volume of generated plaques is significant contrary to their shape. The paper presents a concept of generating a diagnostic parameter that would explicitly enable patient qualification for surgical removal of arterial lesions. The parameter is defined by the ratio of a non-dimensional increase of the hydraulic radius (in relation to the arterial radius) to a relative increase of the plaque area (in relation to the arterial area). The change of this parameter as a function of percentage growth of the area of the solid particles depositing on the arterial wall has a very distinct course – in excess of 60% of the arterial lumen occlusion, there is a drastic deviation from the approximated function and the diagnostic parameter reaches a higher value. A similar course was obtained by drawing a curve of the plaque deposition growth. Deviations from the approximated curve occurred for the same value. The change of the flow radius was of a decreasing nature, which, after exceeding 60%, also started to decrease exponentially.

The physical interpretation of the diagnostic parameter was shown through a simulation of blood flow through a stenosed artery. The numerical studies confirmed the mathematical analysis – upon exceeding of the critical value, vortexes of relatively different areas occurred downstream of the stenosis. The velocity and the area of the vortexes increased along with the increasing stenosis. The acceleration of the vortexes occurring upon exceeding the said value might promptly lead to total occlusion of the arterial lumen. Particularly, through the phenomenon of vortex replication on both sides of the arterial wall, the flow canal becomes curved, which additionally increases the distortions of the flow.

Summarizing the studies, we can state that up to 60% of the flow lumen occlusion, the growth of plaques that follows is of a quasi-linear nature. If the plaque area exceeds 60%, a strong intensification and multiplication of vortexes downstream of the stenosis occurs. The rate of the plaque growth is of a strongly increasing nature and the flow occlusion might grow. Therefore, exceeding the said critical value requires a surgical intervention.

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